Practice papers 1-3 markscheme (pages 746-754)

Paper 1

1. (a) $\int \sin^2 x\, dx = \int \frac{1-\cos(2x)}{2} dx = \frac{x}{2} - \frac{\sin(2x)}{4} + c$ M1A1A1

   (b) $\left[\frac{x}{2} - \frac{\sin(2x)}{4}\right]_0^{\pi} = \frac{\pi}{2}$ M1A1

2. (a) $\log x + \log y = p + q$ (M1)A1

   (b) $\log x^2 - \log y = 2\log x - \log y = 2p - q$ (M1)A1

   (c) $\log\left(x^{\frac{1}{2}}\right) = \frac{1}{2}\log x = \frac{1}{2}p$ (M1)A1

   (d) $\log 100 + \log y = 2 + q$ (M1)A1

3. (a) This is an Arithmetic Progression with common difference 2. (R1)

Let the length of the first piece be *a.*

$290 = \frac{10}{2}(2a + 9 \times 2)$ M1

$290 = 10a + 90 \Rightarrow \quad a = 20$ cm A1

(b) $u_{10} = 20 + 9 \times 2 = 38$ cm M1A1

4. $\int_0^4 \frac{1}{2x+1} \mathrm{d}x = \left[\frac{1}{2}\ln(2x+1)\right]_0^4$ M1A1

$= \frac{1}{2}\ln 9 - 0$ M1A1

$= \ln 3$ A1

5 (i) meaningful A1

(ii) meaningless; $(\mathbf{b}\cdot\mathbf{c})$ is not a vector, so you cannot take the scalar product of $\mathbf{a}$ with it. R1

(iii) meaningful A1

(iv) meaningless; $(\mathbf{b}\cdot\mathbf{c})$ is not a vector, so you cannot take vector product of $\mathbf{a}$ with it. R1

(v) meaningless; both $\mathbf{a}$ and $(\mathbf{b}\cdot\mathbf{c})$ are vectors, so you cannot perform scalar multiplication. R1

(vi) meaningful A1

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6. $\lim\limits_{x\to 0}\dfrac{\tan x - x}{x^3}$ is of indeterminant form $\dfrac{0}{0}$, so can use L'Hopital's rule. M1

$$\lim_{x\to 0}\frac{\tan x - x}{x^3}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\tan x - x\right)}{\frac{d}{dx}\left(x^3\right)}$$

$=\lim\limits_{x\to 0}\dfrac{\sec^2 x - 1}{3x^2}$. Indeterminant form $\dfrac{0}{0}$ $\Rightarrow$ L'Hopital applies. A1

$$\lim_{x\to 0}\frac{\tan x - x}{x^3}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sec^2 x - 1\right)}{\frac{d}{dx}\left(3x^2\right)}$$ M1

$=\lim\limits_{x\to 0}\dfrac{2\sec^2 x\tan x}{6x}$. Indeterminant form $\dfrac{0}{0}$ $\Rightarrow$ L'Hopital applies.A1

$$\lim_{x\to 0}\frac{\tan x - x}{x^3}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(2\sec^2 x\tan x\right)}{\frac{d}{dx}\left(6x\right)}$$ M1

$$=\lim_{x\to 0}\frac{4\sec^2 x\tan^2 x + 2\sec^4 x}{6}=\frac{1}{3}$$ A1A1

7. $f'\left(x\right)=\lim\limits_{h\to 0}\dfrac{\left(x+h\right)^4 - x^4}{h}=\lim\limits_{h\to 0}\dfrac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}$ M1A1M1

$=\lim\limits_{h\to 0}\left(4x^3+6x^2h+4xh^2+h^3\right)=4x^3$ A1A1

8. (a) $p\left(0\right)=9\Rightarrow b=9$ M1A1

$p\left(-1\right)=16\Rightarrow -1-1-a+9=16\Rightarrow a=-9$ M1A1

(b) $p(x)=x^3-x^2-9x+9$ $\quad p\left(1\right)=0\Rightarrow\left(x-1\right)$ is a factor

$p(x)=(x-1)(x^2-9)=(x-1)\left(x-3\right)\left(x+3\right)$ M1A1A1A1

9. (a) (i) -10 (ii) 24 A1A1

10.

(b) (i) graph has been moved 1 unit to the right, so each root has been increased by 1. R1

New sum $= -10 + 4 \times 1 = -6$ A1

(ii) product of roots is $(-1)^4 \frac{a_0}{a_4}$ $a_4 = 1$ M1

$a_o = q(0) = p(-1) = 1 - 10 + 35 - 50 + 24 = 0$ M1

product of roots is 0 A1

Long Questions

10.(a) $D = 36 - 4(8 + k) = 4 - 4k$ M1A1

(b) (i) Concave-up quadratic, always positive, so no roots. R1R1

So $D < 0$ A1

(ii) $D = 4 - 4k < 0 \Rightarrow k > 1$ M1A1

(c) $f(x) = (x+3)^2 + (k-1)$ M1A1

(d) $f(x) > 0 \Rightarrow (x+3)^2 + (k-1) > 0 \Rightarrow k - 1 > 0$ since $(x+3)^2 \geq 0$ for all x.

Hence $k > 1$ as found in (b) (ii). R1M1

(e) for $k = 4$, $f(x) = (x+3)^2 + 3$ so min point is $(-3, 3)$ A1A1

11. Let $P(n)$ be the statement $\sum_{i=1}^{n} i \times i! = (n+1)! - 1$. (M1)

LHS of $P(1)$ is 1. RHS of $P(1)$ is 2! – 1=1. So $P(1)$ is true. M1A1

Assume $P(k)$ is true and attempt to prove $P(k+1)$ M1

$\sum_{i=1}^{k+1} i \times i! = \sum_{i=1}^{k} i \times i! + (k+1) \times (k+1)! = (k+1)! - 1 + (k+1) \times (k+1)!$ M1A1

$= (k+2) \times (k+1)! - 1 = (k+2)! - 1$ as required. M1A1

Since $P(1)$ is true and $P(k)$ true implies $P(k+1)$ true, by the principle of mathematical induction the statement has been proved for all $n \in \mathbb{Z}^+$. R1

(b) $\sum_{i=1}^{n} (i+1-1) \times i! = \sum_{i=1}^{n} (i+1) \times i! - i! = \sum_{i=1}^{n} (i+1)! - \sum_{i=1}^{n} i! = (n+1)! - 1$ as almost all the terms cancel M1M1A1R1

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12(a) $z^4 - 1 = (z^2-1)(z^2+1) = (z-1)(z+1)(z^2+1) = 0$ M1A1

$\Rightarrow z = 1, -1, \mathrm{i},$ or $-\mathrm{i}.$ A1A1

(b) $(r\mathrm{cis}\theta)^8 = 1\mathrm{cis}0 \Rightarrow r^8\mathrm{cis}8\theta = 1\mathrm{cis}0 \Rightarrow r^8 = 1 \Rightarrow r = 1$, M1

$8\theta = 0 \pm 2n\pi, n \in \mathbb{Z} \Rightarrow \theta = \pm\dfrac{n\pi}{4}, n \in \mathbb{Z}$ A1

$z = 1\mathrm{cis}0,\ 1\mathrm{cis}\dfrac{\pi}{4},\ 1\mathrm{cis}\dfrac{\pi}{2},\ 1\mathrm{cis}\dfrac{3\pi}{4},\ 1\mathrm{cis}\pi,\ 1\mathrm{cis}-\dfrac{\pi}{4},\ 1\mathrm{cis}-\dfrac{\pi}{2},\ 1\mathrm{cis}-\dfrac{3\pi}{4}$ A4

(c) $1\mathrm{cis}\dfrac{\pi}{4} = \cos\dfrac{\pi}{4} + \mathrm{i}\sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\mathrm{i}$ M1A1

(d) $\mathrm{i} = e^{\mathrm{i}\frac{\pi}{2}} \Rightarrow \sqrt{\mathrm{i}} = e^{\mathrm{i}\frac{\pi}{4}} = 1\mathrm{cis}\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\mathrm{i}$ by part (c). A1

13.(a) $\displaystyle\int \frac{1}{x(1-x)}dx = \int k\,dt$ M1

$\displaystyle\int \frac{1}{x} + \frac{1}{1-x}dx = kt + c$ M1A1A1

$\ln x - \ln(1-x) = kt + c$ A1

$\ln\left(\dfrac{x}{1-x}\right) = kt + c$ A1

$\dfrac{x}{1-x} = e^{kt+c} = e^c e^{kt} = Ae^{kt}$ A1AG

(b) $\dfrac{\frac{1}{3}}{1-\frac{1}{3}} = Ae^0 \Rightarrow A = \dfrac{1}{2}$ M1A1

(c) $\dfrac{\frac{1}{2}}{1-\frac{1}{2}} = \dfrac{1}{2}e^k \Rightarrow e^k = 2$ M1A1

(d) $\dfrac{x}{1-x} = \dfrac{1}{2}e^{2k} = \dfrac{1}{2}\left(e^k\right)^2 = \dfrac{1}{2}2^2 = 2$ M1A1

$x = 2 - 2x \Rightarrow 3x = 2 \Rightarrow x = \dfrac{2}{3}$ M1A1

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Paper 2

1. $P(X > 7) = 0.69146 \Rightarrow P(X < 7) = 1 - 0.69146 = 0.30854$ A1

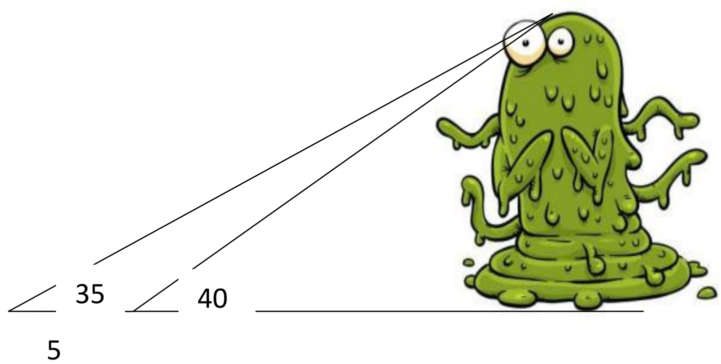
Applying substitution $Z = \dfrac{X-8}{\sigma}$, where $Z \sim N(0,1^2)$, gives M1

$P\left(Z < \dfrac{-1}{\sigma}\right) = 0.30854$ M1

Using inverse normal $\dfrac{-1}{\sigma} = -0.49999...$ M1A1

So $\sigma = 2$ A1

2.(a)



A1

(b) Let the monster's height be *h* and the original distance between Maria and the monster be x.

$\dfrac{h}{x} = \tan 40$, $\dfrac{h}{x+5} = \tan 35$ M1A1

$x \tan 40 = (x+5)\tan 35$ M1

$x(\tan 40 - \tan 35) = 5\tan 35$ A1

$x = \dfrac{5\tan 35}{(\tan 40 - \tan 35)} = 25.206....$ A1

$h = x\tan 40 = 21.2\,\text{m}$ (3 s.f.) A1

[Note: this problem can also be solved using the sine rule.]

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3. (a) $t=0 \Rightarrow v=0$ A1

(b) total distance is $\int_0^3 \left|5\sin\left(t^2\right)\right| dt = 8.51$ m (3 s.f.)

M1A2

(c) displacement is $\int_0^3 5\sin\left(t^2\right) dt = +3.87$ m (3 s.f.)

M1A2

4. Probability that at least one engine is OK on a wing $= 1-0.1\times 0.1 = 0.99$ M1A1

Probability that both wings are OK $= 0.99^2 = 0.9801(4\text{ d.p.})$

M1A1

5 (a) $\int_{-1}^{1} e^{x^2} dx = 2.93(3sf)$ M1A2

(b) $\pi\int_{-1}^{1} (e^{x^2})^2 dx = 14.9(3sf)$ M1A2

6. $\dfrac{5}{\sin C} = \dfrac{4}{\sin 40} \Rightarrow \sin C = \dfrac{5\sin 40}{4} \Rightarrow C = 53.46...or\ 126.54..$ M1A1

$B = 86.54...or\ 13.46...$ A1

Area $= \frac{1}{2}\times 5\times 4\sin 86.54...or\ \frac{1}{2}\times 5\times 4\sin 13.46...$ $9.98\ or\ 2.33$ M1A1

So $B = 13.46...$

$\dfrac{AC}{\sin 13.46...} = \dfrac{4}{\sin 40}$ $AC = 1.45$ (3 s.f.) M1A1

7. (a) By symmetry the mode occurs at $X = n$ R1

(b) $P\left(X=n\right) = \dfrac{35}{128} \Rightarrow n = 4$ (by use of "table" command on GDC) A2

8. (a) $A = \pi r^2$ $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$ M1A1

$\left.\dfrac{dA}{dt}\right|_{r=5} = 2\pi\times 5\times 0.1 = 3.14\left(3\text{ s.f.}\right)$ km^2/h M1A1

(b) $6 = \dfrac{dA}{dt} = 2\pi r\times 0.1 \Rightarrow r = \dfrac{6}{0.2\pi} = 9.55\text{ km }\left(3\text{ s.f.}\right)$ M1A1

9.(a) $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 4 & -1 & -1 & 6 \\ 5 & 1 & k & 12 \end{array}\right)$ $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -9 & -13 & -22 \\ 0 & -9 & k-15 & -23 \end{array}\right)$ $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -9 & -13 & -22 \\ 0 & 0 & k-2 & -1 \end{array}\right)$

So $k=2$ for no solutions. M1A1A1A1

(b) $x=2, y=1, z=1$ A1A1A1

Long Questions

10. (a)

$\vec{v}\cdot\vec{w}=1\times 2+2\times(-1)+3\times 2=6$

M1A1

Also, $\vec{v}\cdot\vec{w}=|v||w|\cos\theta$

$=\sqrt{1^2+2^2+3^2}\sqrt{2^2+(-1)^2+2^2}\cos\theta$

$=\sqrt{14}\times 3\cos\theta$ A1

$\cos\theta=\dfrac{2}{\sqrt{14}}\Rightarrow\theta=57.7^\circ\,(3sf)$ A1A1

(b) The planes are perpendicular to $\vec{v}$ and $\vec{w}$ R1

Angle between planes equals angle between perpendiculars= $57.7^\circ\,(3sf)$ R1A1

(c) line is parallel to $\vec{w}$, angle between perpendicular to plane and the line is 57.7°

R1R1

So angle between plane and line is $90^\circ-57.7^\circ=32.3^\circ\,(3sf)$. A1

(d) Point on the line has the form $x=2\lambda+5, y=-\lambda-6, z=2\lambda+5$ M1A1

Require $2\lambda+5-2\lambda-12+6\lambda+15+4=0\Rightarrow\lambda=-2$ M1A1

Point is $(1,-4,1)$ A1

11. (a) (i) $x=5$ (ii) $y=3$ A1A1

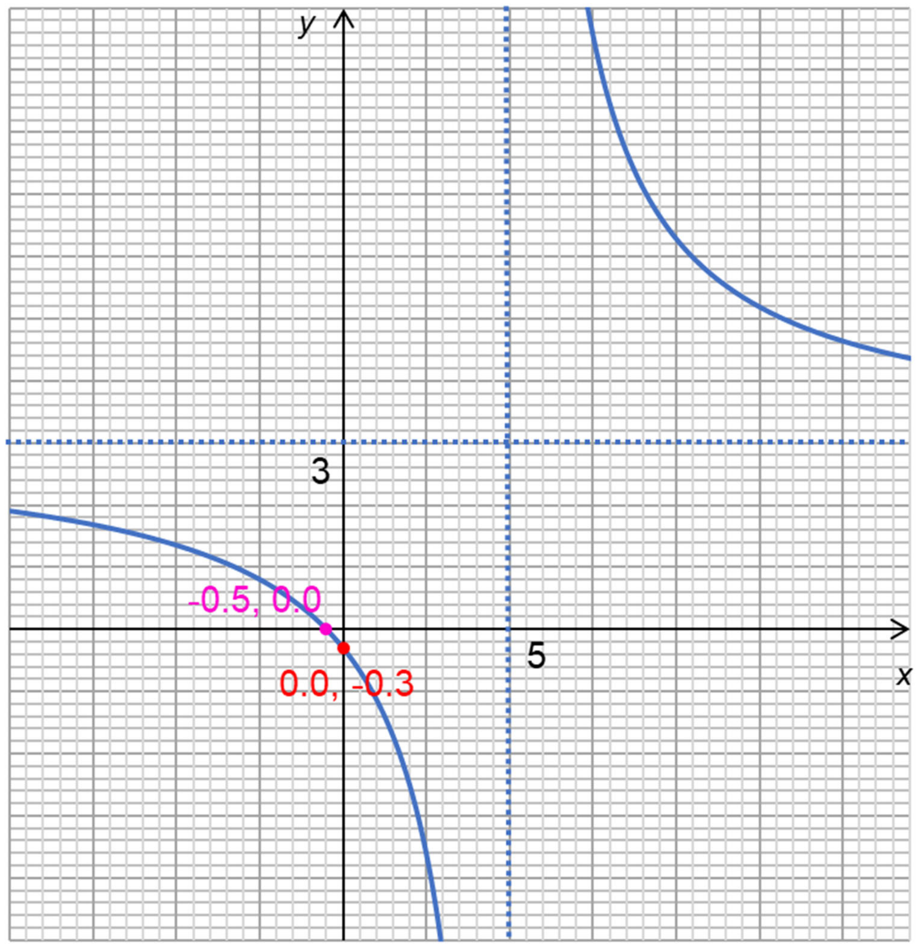
(b) (ii) $\left(0,-\frac{3}{10}\right)$ (ii) $\left(-\frac{1}{2},0\right)$

A1A1

(c) (i) . $f'(x)=\dfrac{6(2x-10)-(6x+3)2}{(2x-10)^2}=-\dfrac{66}{(2x-10)^2}$ M1A1

(ii) $f'(x)$ is always negative so graph is always decreasing R1A1

(d)



A3

(e) $y = -\frac{11}{6}x + \frac{7}{6}$ (3 s.f.) A1A1

12. (a) $2 \times 0.8^5 = 0.655(3sf)$ m M1A1

(b) solving $2 \times 0.8^n < 0.25$ gives $n = 10$ M1A2

(c) heights of bounce form an infinite GP with $a = 2, r = \frac{4}{5}$. M1

$S_\infty = \frac{2}{1-\frac{4}{5}} = 10$ M1A1

Total distance ball travels is twice each height, except the first one. R1

So the total distance is $2S_\infty - 2 = 2 \times 10 - 2 = 18$ m A2

(d) times also form an infinite GP R1

$2 = 5t^2$ gives first term of $t_1 = \sqrt{\frac{2}{5}}$. A1

$t = \sqrt{\frac{s}{5}} \Rightarrow$ Common ratio for *t* is $\frac{t_2}{t_1} = \frac{\sqrt{\frac{s_2}{5}}}{\sqrt{\frac{s_1}{5}}} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{4}{5}}$ A1

Total time is twice the time it takes to travel each distance *s*, except for the first one.

So the total distance is $2S_\infty - \sqrt{\frac{2}{5}}$ R1

$= 2\frac{\sqrt{\frac{2}{5}}}{1-\sqrt{\frac{4}{5}}} - \sqrt{\frac{2}{5}} = 11.3\text{ s (3 s.f.)}$ M1A2

13. (a) $\int_{-\frac{1}{2}}^{\frac{1}{2}} a\cos\pi x\, dx = 1 \Rightarrow \left[\frac{a}{\pi}\sin\pi x\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \left(\frac{a}{\pi}\right) - \left(-\frac{a}{\pi}\right) = 1 \Rightarrow a = \frac{\pi}{2}$ M1A1A1

(b) (i) $\mu = \int_{-\frac{1}{2}}^{\frac{1}{2}} x\frac{\pi}{2}\cos\pi x\, dx = 0$

M1A1

(ii) $\sigma^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2\frac{\pi}{2}\cos\pi x\, dx - 0^2 = 0.0474\,(3sf)$ M1A1

(c) $P\left(x < \frac{1}{4} \middle| x > 0\right) = \frac{P\left(0 < x < \frac{1}{4}\right)}{P(x > 0)} = \frac{0.3535...}{0.5} = 0.707\,(3sf)$ M1A1A1

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Paper 3

1. (a)(i) arithmetic progression A1

(ii) $u_n = u_1 + (n-1)b$ A1

(iii) $S_n = \frac{n}{2}\left(2u_1 + (n-1)b\right)$ A1

(b)(i) Geometric progression A1

(ii) $u_n = u_1 a^{n-1}$ A1

(iii) $S_n = u_1 \frac{(a^n - 1)}{(a-1)}$ A1

(c) (i) $u_2 = 3,$, $u_3 = 7$, $u_4 = 15$, $u_5 = 31$. A1A1

(ii) $u_n = 2^n - 1$ A1

(iii) Let $P(n)$ be the statement $u_n = 2^n - 1$.

$P(1) = 2^1 - 1 = 1$ which is true, since the question gives $u_1 = 1$. A1

Assume the result for $P(k)$ and attempt to prove for $P(k+1)$. M1

By the recurrence relation, $u_{k+1} = 2u_k + 1 = 2\left(2^k - 1\right) + 1 = 2^{k+1} - 1$ as required

M1A1A1

Since $P(1)$ is true and $P(k)$ true implies $P(k+1)$ is also true. Hence, by the principle of mathematical induction, $P(n)$ is true for all $n \in \mathbb{Z}^+$. R1

(d)(i) $u_{n+1} + c = a(u_n + c) \Rightarrow u_{n+1} = au_n + ac - c \Rightarrow ac - c = b$ M1A1

$\Rightarrow c = \frac{b}{a-1}$ A1

(ii) $v_{n+1} = u_{n+1} + c$

$= \left(au_n + ac - c\right) + c$

$= a\left(u_n + c\right)$

$= av_n$

Hence, $v_{n+1} = av_n$, which is a geometric progression A1

(iii) $v_n = a^{n-1}v_1$ A1

(iv) $u_n + c = a^{n-1}(u_1 + c) \Rightarrow u_n + \frac{b}{a-1} = a^{n-1}u_1 + a^{n-1}\frac{b}{a-1}$ M1A1

$\Rightarrow u_n = a^{n-1}u_1 + b\frac{\left(a^{n-1} - 1\right)}{a-1}$ A1

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(e) General formula gives $2^{n-1}\times 1+1\dfrac{\left(2^{n-1}-1\right)}{2-1}=2^{n-1}+2^{n-1}-1=2^{n}-1$ giving agreement

M1A1

2.

(a) $0.881(3sf)$ A2

(b) $\arcsin x+c$ A1

(c) Let $x=\sin\theta$, integral becomes $\int\dfrac{1}{\sqrt{1-\sin^{2}\theta}}\cos\theta\, d\theta$ M1A1

$=\int 1\, d\theta=\theta+c=\arcsin x+c$ A1A1

(d) $(\cosh x)^{2}-(\sinh x)^{2}=\dfrac{e^{2x}+2+e^{-2x}}{4}-\dfrac{e^{2x}-2+e^{-2x}}{4}=\dfrac{4}{4}=1$ M1A1

(e) (i) $\dfrac{d\sinh x}{dx}=\dfrac{e^{x}+e^{-x}}{2}=\cosh x$ M1A1

(ii) $\dfrac{d\cosh x}{dx}=\dfrac{e^{x}-e^{-x}}{2}=\sinh x$ M1A1

(f) Let $y=\sinh x=\dfrac{e^{x}-e^{-x}}{2}$.

Then inverse function will be given by making *y* the subject of $x=\dfrac{e^{y}-e^{-y}}{2}$. M1

$2x=e^{y}-e^{-y}\Rightarrow 2xe^{y}=\left(e^{y}\right)^{2}-1\Rightarrow\left(e^{y}\right)^{2}-2x\left(e^{y}\right)-1=0$ M1A1

$e^{y}=\dfrac{2x\pm\sqrt{4x^{2}+4}}{2}=x\pm\sqrt{x^{2}+1}$ A1

Since $e^{y}>0$ we must take the positive square root. R1

$y=\operatorname{arsinh}x=\ln\left(x+\sqrt{x^{2}+1}\right)$ AG

(g) $I=\int\dfrac{1}{\sqrt{1+\sinh^{2}u}}\cosh u\, du=\int 1\, du=\operatorname{arsinh}x+c=\ln\left(x+\sqrt{x^{2}+1}\right)+c$

M1A1A1A1

(h) $\int_{0}^{1}\dfrac{1}{\sqrt{1+x^{2}}}dx=\left[\ln\left(x+\sqrt{x^{2}+1}\right)\right]_{0}^{1}=\ln\left(1+\sqrt{2}\right)$ which agrees with 0.881 M1A1

(i) $I=\int\dfrac{1}{\sqrt{1+\tan^{2}\theta}}\sec^{2}\theta\, d\theta=\int\sec\theta\, d\theta$ M1A1

(j) $\int\sec\theta\, d\theta=\int\dfrac{\sec^{2}\theta+\sec\theta\tan\theta}{\tan\theta+\sec\theta}d\theta=\ln\left(\tan\theta+\sec\theta\right)+c$ M1A1

(k) $\tan\theta=x$, $1+\tan^{2}\theta=\sec^{2}\theta$, $\sec\theta=\sqrt{1+\tan^{2}\theta}=\sqrt{1+x^{2}}$

So $\ln(\tan\theta+\sec\theta)+c=\ln\left(x+\sqrt{1+x^2}\right)+c$ as before

M1A1

(l) $f'(x)=1+x\left(1+x^2\right)^{-\frac{1}{2}}$ M1A1

(m) $I=\int\frac{1}{\sqrt{1+x^2}}\frac{\left(1+x\left(1+x^2\right)^{-\frac{1}{2}}\right)}{\left(1+x\left(1+x^2\right)^{-\frac{1}{2}}\right)}dx=\int\frac{f'(x)}{\sqrt{1+x^2}+x}dx=\int\frac{f'(x)}{f(x)}dx$ M1A1

$=\ln(f(x))+c=\ln\left(x+\sqrt{1+x^2}\right)+c$ as before A1